Welfare Egalitarianism Under Uncertainty*

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Abstract

This paper studies ranking allocations in economic environments where the future endowments are state contingent. The goal is to establish social orderings based on individual preferences that are both ordinal and noncomparable. To achieve this, we determine the certainty equivalent welfare levels for each individual, representing the welfare levels that render them indifferent to their initial endowments. These individual welfares are then ranked using the leximin ordering. By incorporating efficiency, equity, and robustness conditions, we define and characterize the Certainty Equivalent Welfare Maximin Ordering.

Keywords: social choice, fairness, uncertainty, ex-ante egalitarianism, maximin, state-contingent endowment

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1 Introduction

The assessment of social situations in the presence of risk and uncertainty has been a topic of considerable debate within the field of welfare economics, dating back

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to Harsanyi (1955). In his renowned Social Aggregation Theorem, Harsanyi demonstrates that if individuals and the social planner have expected utility-consistent preferences, then the Pareto principle dictates that the social welfare be affine with respect to individual utilities. However, this utilitarian approach to social welfare has faced criticism from various scholars due to its perceived indifference towards the equitable distribution of welfare.[1]

Harsanyi's utilitarian characterization of social welfare gives rise to a significant challenge regarding the reconciliation of social rationality, efficiency, and fairness. To incorporate more egalitarian social welfare orderings, one must either relax social rationality or the strict application of the Pareto principle. Diamond (1967) and Epstein and Segal (1992) adopt an "ex-ante" approach, relaxing social rationality by allowing for an inequality-averse social welfare ordering that maintains the Pareto principle. Specifically, Epstein and Segal (1992) define a quadratic social welfare function that incorporates a preference for randomization of lotteries. In contrast, Adler and Sanchrico (2006) take an "ex-post" approach, maintaining social rationality while relaxing the Pareto principle. They characterize a non-Paretian social welfare function that is both inequality-averse and expected utility consistent. As a compromise between these two approaches, Fleurbaey (2010) applied Pareto principles for riskless environments, and for risky environments without inequalities ex-post. Accordingly, he singled out social welfare orderings in the form of "Expected Equally Distributed Equivalent" by Pareto axioms and a weaker social rationality condition called "statewise dominance".

In our paper, we focus on an environment where the future incomes of individuals are subject to uncertainty. Our primary objective is to develop a fair method for aggregating individual preferences into a social preference within this risky setting. To depart from Harsanyi's utilitarian characterization, we adopt an alternative approach known as "Fair Social Choice Theory" (FSCT), as outlined in Maniquet

[1] For further philosophical perspectives on this matter, see Diamond (1967), Rawls (2020), Sen (1970, 1979).

and Fleurbaey (2011). Within this theory, egalitarian social choice orderings can be characterized by relaxing Arrow's Independence of Irrelevant Axiom and incorporating equity axioms from the fair allocation literature. By utilizing the Arrovian framework, which allows for the ranking of any two social allocations, this approach provides a fine-grained ranking. In contrast to the fair allocation literature, which typically employs a two-tier ranking system (optimal versus non-optimal allocations), the Fair Social Choice Theory (FSCT) approach offers distinct advantages, particularly when it comes to implementation challenges. In certain cases, policymakers may need to choose among non-optimal allocations due to issues like incentive constraints arising from asymmetric information or concerns regarding the status quo (e.g., linear taxation).[2]

Additionally, FSCT employs ordinal preferences and incorporates interpersonal comparisons based on preferences over resources. This approach aligns with Rawls (1971) and Sen (1992), who argue that utility comparisons involve value judgments and are therefore not comparable across individuals. As a result, interpersonal comparisons should be rooted in a resource metric.

Fair Social Choice Theory provides two prominent social choice rules, both of which are based on the egalitarian idea. The first rule is "Egalitarian Equivalent" ordering due to Pazner and Schmeidler (1978). And the second rule is "Egalitarian Walrasian" ordering which is based on Walrasian equilibrium outcome after the initial endowment is split equally. In our paper, we use the first approach to characterize fair (egalitarian) social orderings in our risky environment. Moreover, we do not restrict ourselves to the expected utility domain. Our preference domain is as large as possible including complete, transitive, convex, continuous, and monotone preferences over state contingent goods. We define welfare as the "certainty equivalent" allocation and take the maximin ordering. Under this definition, social welfare

[2]Notably, various papers have utilized the Fair Social Choice Theory framework, including works by Fleurbaey and Maniquet (2005, 2006, 2008, 2011), Maniquet (2008), and Maniquet and Sprumont (2004, 2005, 2008).

is the welfare of the worst-off individual in the economy. In the literature, there are various characterizations of maximin social welfare orderings under risk and uncertainty. Under the Expected Utility consistent preference domain, Fleurbaey and Maniquet (2011, Theorem 6.1) characterized certainty equivalent maximin ordering by Pareto, transfer, and separability axioms. Miyagishima (2016), on the other hand, provided a different characterization of certainty equivalent maximin ordering in his corresponding domain.

Our paper differs from the rest of the literature, by taking the preference domain as large as possible. Our transfer axiom is inspired by Maniquet and Sprumont (2004), where welfare egalitarianism is defined in the economies with one private good and one partially excludable nonrival good. They define an individual's welfare as the amount of the nonrival good which leaves him indifferent to his initial consumption bundle. This allows them to rank bundles by the leximin criterion, They characterize this nonrival maximin ordering by Unanimous Indifference, Responsiveness, and Free Lunch Aversion axioms. Our paper can be regarded as an extension of Maniquet and Sprumont (2004) to economies with state-contingent endowments. The natural way of defining welfare in this framework is the "riskless" allocation, namely certainty equivalent allocation over state contingent endowments. The main contribution of this paper is our definition of an equity criterion ensuring some form of aversion to income inequality where inequality is defined as two individuals being affected by an event in opposite directions. One can find this axiom quite compelling for catastrophic risks, such as natural disasters (earthquakes, hurricanes, etc.), where it is socially undesirable for some individuals to benefit while others are harmed. This axiom, combined with the efficiency and robustness conditions, leads to a social ordering with an infinite aversion to inequality – a maximin ordering.

The rest of the paper is organized as follows: In Section 2, we introduce the axioms and the model. We state the results in Section 3. Finally, section 4 concludes with possible directions for future research.

2 Preliminaries

Consider a finite set of individuals N with $|N| \geq 2$. S is a finite set of distinct states of nature, with $|S| \geq 2$. $\Omega \in (\mathbb{R}_{+}^{S})^{N}$ denotes the *social endowment* of the state contingent goods. The consumption of individual $i \in N$ at state $s \in S$ is denoted as $z_{is} \in \mathbb{R}_{+}$. Each individual $i \in N$ has an ex-ante, state-independent preference relation $R_i \in \mathcal{R}$, a complete and transitive binary relation over the state contingent endowment, which is also convex, continuous, and strictly increasing in each state contingent good. A *social preference profile* is denoted as $R = (R_i)_{i \in N} \in \mathcal{R}^N$. An *economy* is defined as a quadruple $E = (N, S, \Omega, R) \in \mathcal{E}$. An *allocation* is a vector of $z_N = (z_i)_{i \in N} \in (\mathbb{R}_{+}^{S})^{N}$. An allocation is feasible if $\sum z_i \leq \Omega$. The set of feasible allocations is denoted as $Z(E)$. The *Upper contour set* of R_i at z_i is denoted as $B(R_i, z_i) = \{z_i' \in \mathbb{R}_{+}^{s} | z_i' R_i z_i\}$. For each $R_i \in \mathcal{R}$ and for each $z_i \in \mathbb{R}_{+}^{S}$, there is a unique level of $c(R_i, z_i) \in \mathbb{R}_{+}$ such that $z_i I_i c(R_i, z_i)\mathbf{1}_s$ where $\mathbf{1}_s = (1, ..., 1) \in \mathbb{R}_{+}^{S}$. Certainty equivalent welfare level of individual i with preference profile R at z_i is given by $c(R_i, z_i)$.

For the sake of exposition, we use two-state world for the rest of the paper. This is by no means a restriction as for any $S \geq 2$ we can represent $S - 1$ states as a projection to one state. Accordingly, we write $z_i = (x_i, y_i)$ where x_i denotes individual i's endowment for state 1 and y_i denotes individual i's endowment for state 2. Next, social preference is found by applying the leximin ordering to the individual certainty equivalent welfare levels. We provide an axiomatization of this particular certainty equivalent maximin ordering.

First, *Unanimous Indifference* condition says that two allocations that leave all the individuals indifferent should be deemed socially equivalent. This is a weaker condition than Pareto, and it is clearly satisfied by certainty equivalent leximin ordering. Second the *Responsiveness* condition ensures that social ordering is preserved if better sets for all individuals shrink for the better allocation, and they expand for the worse allocation. And finally, *Aversion to Attendant Gains* is the equity con-

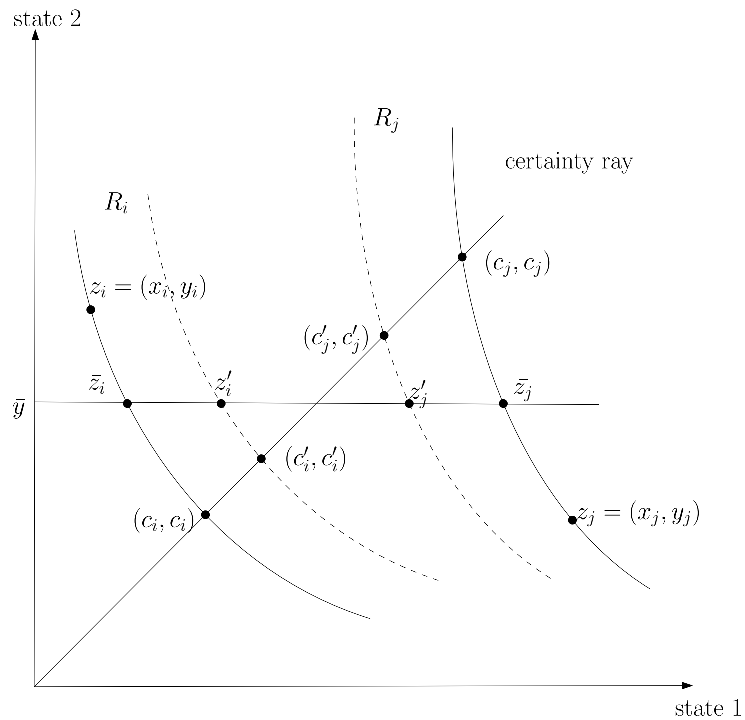

Figure 1: CE Leximin Ordering satisfies AAG.

dition requiring a transfer between two agents as a social improvement, as long as they have the same endowment under one event and the transfer is done under the event in which the endowment of two agents lie on the opposite sides of the certainty equivalent line provided that their orientation with respect to certainty ray remains the same. Figure 1 illustrates how Certainty Equivalent Leximin ordering satisfies the Aversion to Attendant Gains condition. By Unanimous Indifference, one can move along the indifference curve such that $(z_{1,}z_2)\mathbf{I}(E)(\bar{z}_{1,}\bar{z}_2)$. And by Aversion to Attendant Gains, we have $(z'_{1,}z'_2)\mathbf{R}(E)(\bar{z}_{1,}\bar{z}_2)$ as $\min(c'_i, c'_j) = c'_i > c_i = \min(c_i, c_j)$.

Now, we will turn to the formal model. The first axiom captures the minimum efficiency condition. Unanimous Indifference requires social preferences to agree with individual preferences.

Definition 1. Unanimous Indifference (UI): *Let* $E = (N, S, \Omega, R) \in \mathcal{E}$ *be given and let* $z_N, z'_N \in Z(E)$*, If* $z_i I_i z'_i$*, for all* $i \in N$*, then* $z_N \mathbf{I}(E) z'_N$

In the next section, we show that this axiom, combined with the Responsiveness and Aversion to Attendant Gains axioms, implies stronger efficiency conditions, Unanimous Preference, and Unanimous Strict Preference.

Definition 2. Unanimous Preference (UP): *Let* $E = (N, S, \Omega, R) \in \mathcal{E}$ *be given an let* $z_N, z'_N \in Z(E)$*. If* $z_i R_i z'_i$*, for all* $i \in N$*, then* $z_N \mathbf{R}(E) z'_N$*.*

Definition 3. Unanimous Strict Preference (USP): *Let* $E = (N, S, \Omega, R) \in \mathcal{E}$ *be given and let* $z_N, z'_N \in Z(E)$*. If* $z_i P_i z'_i$*, for all* $i \in N$*, then* $z_N \mathbf{P}(E) z'_N$*.*

The second axiom presents a robustness condition that can also be seen as an independence axiom. It is borrowed from Fleurbaey and Maniquet (1996). Say an allocation z_N is socially preferred to another allocation z'_N. The Responsiveness condition ensures that social preference is preserved if better sets of all the individuals shrink for the "better" allocation and they expand for the "worse" allocation.

Definition 4. Responsiveness (R): *Let* $E = (N, A, \Omega, R) \in \mathcal{E}$ *and* $E' = (N, A, \Omega, R') \in \mathcal{E}$ *be given. Let* $z_N, z'_N \in Z(E)$*. Let* $B(R'_i, z_i) \subseteq B(R_i, z_i)$ *and* $B(R'_i, z'_i) \supseteq B(R_i, z'_i)$ *for all* $i \in N$*, then* $\{z_N \mathbf{R}(E) z'_N\} \Rightarrow \{z_N \mathbf{R}(E') z'_N\}$ *and* $\{z_N \mathbf{P}(E) z'_N\} \Rightarrow \{z_N \mathbf{P}(E') z'_N\}$

Finally, we define an equity criterion relevant to our framework which is inspired by Free Lunch Aversion Axiom introduced by Maniquet and Sprumont (2004).[3] It is a fairly minimal inequality aversion condition whose ethical justification was presented in the introduction. Aversion to Attendant Gains condition says that if two individuals face the risk of one unexpected event in opposite directions, then reducing the gap of that risk by transfer improves social welfare, provided that the orientation with respect to certainty ray would not change after transfer. This axiom is clearly weaker than Pigou-Dalton transfer, and unlike Pigou-Dalton transfer it does not contradict with the efficiency.[4]

[3] Free Lunch Aversion axiom of Maniquet and Sprumont (2004) was in turn inspired by the No Private Transfers axiom due to Moulin (1987).

[4] See Theorem 2.1. Fleurbaey and Maniquet (2011).

Definition 5. Aversion to the Attendant Gains (AAG) with respect to state s: *Let* $E = (N, S, \Omega, R) \in \mathcal{E}$ *be given. Let* $z_N, z'_N \in Z(E)$ *such that there exist* $s \in S$ *and* $i, j \in N$ *with* $z_{is} = z_{js}$ *and there exist* $t \in S$ *and* $\Delta > 0$ *such that* $z_{it} < z_{it} + \Delta = z'_{it} < z_{is} = z_{js} < z'_{jt} = z_{jt} - \Delta < z_{jt}$ *and* $z_{ks} = z'_{ks}$ *for all* $k \neq i, j$ *and for all* $s \in S$. *Then,* $z'_N \mathbf{P}(E) z_N$.

3 The Results

A social ordering is in the form of certainty equivalent maximin, if the ordering of two social allocations is obtained according to the maximin ordering of certainty equivalent welfare levels. That is, for any $R \in \mathcal{R}^N$ and for any $z_N, z'_N \in (\mathbb{R}^S_+)^N$

$$\min_{i \in N} c(R_i, z_i) > \min_{i \in N} c(R_i, z'_i) \Longrightarrow z_N \mathbf{P}(E) z'_N$$

Leximin ordering is an eminent example of maximin ordering. Let $\succsim_L$ denote the usual leximin ordering[5] on $(\mathbb{R}^S_+)^N$. Certainty Equivalent Welfare Leximin Ordering $\mathbf{R}^C$ ranks the vectors of certainty equivalent welfare levels by applying leximin ordering. For any $R \in \mathcal{R}^N$ and for any $z_N, z'_N \in (\mathbb{R}^S_+)^N$

$$z_N \mathbf{R}^C(E) z'_N \Longleftrightarrow (c(R_i, z_i))_{i \in N} \succsim_L (c(R_i, z'_i))_{i \in N}$$

Before going into our characterization theorem, we will state two lemmas. It is important to note that Unanimous Indifference is a fairly minimal condition of efficiency. The next two lemmas show that stronger efficiency criteria, such as Unanimous Preference and Unanimous Strict Preference, could be obtained by adding Responsiveness and Aversion to the Attendant Gains conditions. Our proofs of the next two lemmas and the main theorem are closely linked with Maniquet and Sprumont (2004).

Lemma 1. *If a social ordering satisfies Unanimous Indifference and Responsiveness,*

[5]For two vectors $u_N, v_N \in \mathbb{R}^N_+$, we have $u_N \succsim_L v_N$ if the smallest component of u_N is larger than v_N. If they are equal the next smallest component is compared, and so on.

then it satisfies Unanimous Preference.

Proof. Suppose $\mathbf{R}$ satisfies Unanimous Indifference and Responsiveness. To get a contradiction, assume that $\mathbf{R}$ fails Unanimous Preference. That is, there exist $R \in \mathcal{R}^N$ and two social allocations $z_N^1, z_N^2 \in Z(E)$ with $z_N^1 \mathbf{P}(E) z_N^2$ and there exists $M \subseteq N$ such that $z_i^2 P_i z_i^1$, for all $i \in M$ and $z_j^2 I_j z_j^1$, for all $j \in N \backslash M$. Without loss of generality assume that $M = \{i\}$.[6]

As shown in Figure 2, choose z_i^3 such that $z_i^3 I_i z_i^1$ and $y_i^3 > y_i^1, y_i^2$. Let C be the convex hull of $\{(x_i, y_i) \in B(R_i, z_i^1) \mid y_i^1 \geq y_i^3\} \cup B(R_i, z_i^2)$ and let $\partial C = \{(x_i, y_i) \in C \mid ((x_i', y_i') = (x_i, y_i)$, for all $(x_i, y_i) \in C$ such that $x_i' \leq x_i$ and $y_i' \leq y_i\}$. So, there exists $z_i^4 \in \partial C$ such that $z_i^4 I_i z_i^2$. By Unanimous Indifference, $(z_i^3, z_{-i}^1)\mathbf{P}(E)(z_i^4, z_{-i}^2)$. Now we can construct $R_i' \in \Re$ such that $B(R_i', z_i^3) = C$. By continuity and strict monotonicity of the preferences there exists $z_i^4 \in \partial C$ such that $z_i^4 I_i' z_i^3$. Since $B(R_i', z_i^3) \subseteq B(R_i, z_i^3)$ and $B(R_i', z_i^4) \supseteq B(R_i, z_i^4)$, by Responsiveness we get $(z_i^3, z_{-i}^1)\mathbf{P}(E')(z_i^4, z_{-i}^2)$, which contradicts with the Unanimous Indifference. □

Lemma 2. *If a social ordering satisfies Unanimous Preference and Aversion to the Attendant Gains, then it satisfies Unanimous Strict Preference.*

Proof. Suppose $\mathbf{R}$ satisfies Unanimous Preference and Aversion to the Attendant Gains. To get a contradiction, assume that $\mathbf{R}$ fails Unanimous Strict Preference. That is, there exist $R \in \mathcal{R}^N$ and two social allocations $z_N, \tilde{z}_N \in Z(E)$ with $z_N \mathbf{R}(E) \tilde{z}_N$ such that $\tilde{z}_i P_i z_i$ for all $i \in N$. Without loss of generality, assume that $c(R_1, z_1) \geq c(R_i, z_i)$, for all $i \in N$. Therefore $c(R_1, \tilde{z}_1) \geq c(R_i, z_i)$, for all $i \in N$. As shown in Figure 3, we can choose $\hat{z}_1 = (\hat{x}_1, \bar{y})$ and $\hat{z}_2 = (\hat{x}_2, \bar{y})$.

Then there exists $\Delta > 0$ such that $\hat{x}_2 + \Delta \leq y \leq \hat{x}_1 - \Delta$ and $(\hat{x}_1, y) P_1 (\hat{x}_1 - \Delta, y)$ and $(\hat{x}_2 + \Delta, y) P_2 (\hat{x}_2, y)$.

By Aversion to the Attendant Gains, $((\hat{x}_1 - \Delta, y), (\hat{x}_2 + \Delta, y), z_{-12})\mathbf{P}(E)((\hat{x}_1, y), (\hat{x}_2, y), z_{-12})$.

By Unanimous Indifference, $((\hat{x}_1, y), (\tilde{x}_2, y), z_{-12})\mathbf{I}(E)(\tilde{z}_1, z_2, z_{-12})$.

And by Unanimous Preference $(\tilde{z}_1, z_2, z_{-12})\mathbf{R}(E)(z_1, z_2, z_{-12})$.

[6] For $|M| \geq 2$, construct a sequence of $\{z(t)\}_{t=0}^{t=|N|}$ where $z_j(t) = z_j^2$ for $j \leq t$ and z_j^1 otherwise. Because R is transitive, there exists some $t \in \{1, ..., |N|\}$ such that $z(t-1)\mathbf{P}(R)z(t)$.

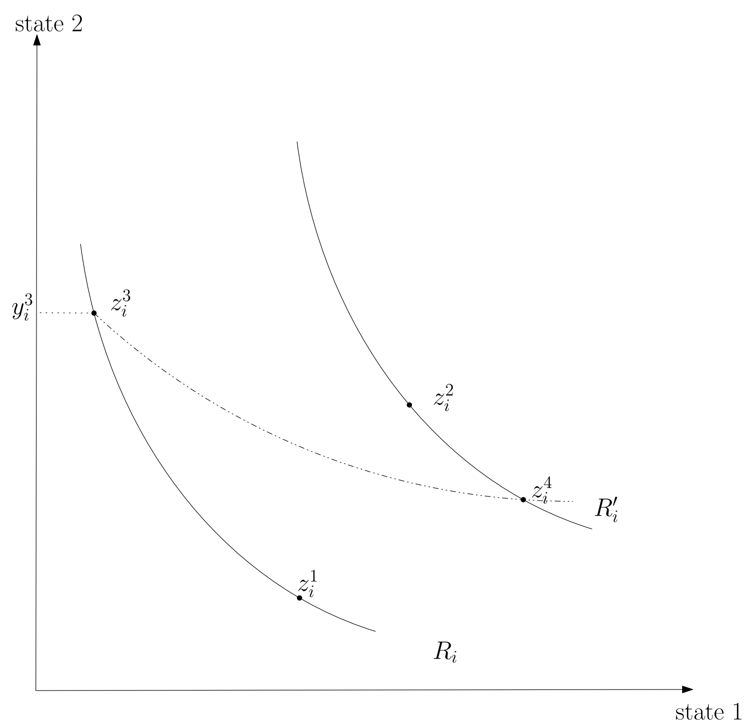

Figure 2: UI and R implies UP.

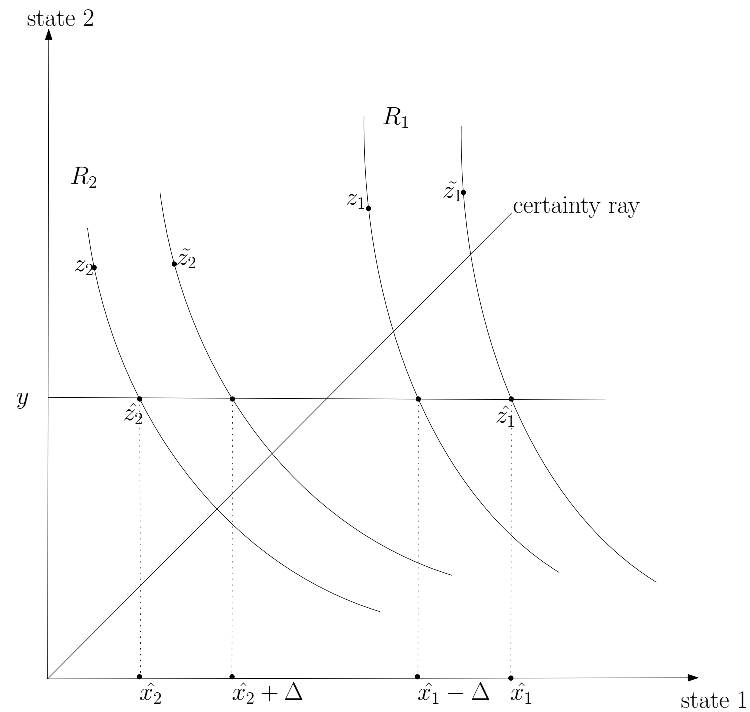

Figure 3: UR and AAG implies USP.

Since $z_N \mathbf{R}(E) \tilde{z}_N$ we get $((\hat{x}_1 - \Delta, y), (\hat{x}_2 + \Delta, y), z_{-12}) \mathbf{P}(E) (\tilde{z}_1, \tilde{z}_2, \tilde{z}_{-12})$, which contradicts with the Unanimous Preference. □

The previous two lemmas show that social preferences follow, not only for indifference to individual preferences but also follow for weak and strict preferences. Now we are ready to state our main characterization theorem.

Theorem 1. *The Certainty Equivalent Leximin ordering* $\mathbf{R}^C$ *satisfies Unanimous Indifference, Responsiveness and Aversion to Attendant Gains. Conversely, every social ordering* $\mathbf{R}$ *satisfying Unanimous Indifference, Responsiveness and Aversion to Attendant Gains is in the form of certainty equivalent maximin.*

Proof. First, we will show that Certainty Equivalent Leximin ordering $\mathbf{R}^C$ satisfies Unanimous Indifference, Responsiveness, and Aversion to the Attendant Gains.

Let $R \in \mathcal{R}^N$ and $z_N, z'_N \in Z(E)$ such that $z_i I_i z'_i$ for all $i \in N$. So $c(R_i, z_i) = c(R_i, z'_i)$ for all $i \in N$. Therefore $z_N \mathbf{I}(E) z'_N$. So Unanimious Indifference holds.

To show that Responsiveness is satisfied assume that $z_N \mathbf{R}(E) z'_N$ with $B(R'_i, z_i) \subseteq B(R_i, z_i)$ and $B(R'_i, z'_i) \supseteq B(R_i, z'_i)$ for all $i \in N$. Then $c(R'_i, z_i) \geq c(R_i, z_i)$ and $c(R'_i, z'_i) \leq c(R_i, z'_i)$, for all $i \in N$. So $z_N \mathbf{R}(E') z'_N$. Hence Responsiveness holds.

And to check Aversion to the Attendant Gains, let $i, j \in N$ and assume that $z_i = (x_i, y)$; $z_j = (x_j, y)$ where $x_i > y$ and $x_j < y$ and $x_j < x'_j = x_j + \Delta \leq y \leq x_i - \Delta = x'_i < x_i$. Further assume that $z_{-ij} = z'_{ij}$.

Then $c(R_i, (x'_i, y)) < c(R_i, z_i)$ and $c(R_j, (x'_j, y)) > c(R_j, z_j)$

So $(c(R_i, z'_i))_{i \in N} \succsim_L (c(R_i, z_i))_{i \in N}$ which implies $z' \mathbf{P}(E) z$. Thus Aversion to the Attendant Gains holds as well.

Now we will prove that a social ordering satisfying Unanimous Indifference, Responsiveness and Aversion to the Attendant Gains has to be in the form of certainty equivalent maximin.

To get a contradiction, suppose that there exists $R \in \mathcal{R}^N$ and $z_N, z'_N \in Z(E)$ such that $\min_{i \in N} c(R_i, z_i) < \min_{i \in N} c(R_i, z'_i)$ yet $z_N \mathbf{R}(E) z'_N$.

So $c(R_i, z_i) \leq \min_{k \in N} c(R_k, z'_k) \leq c(R_j, z_j)$ for all $i \in M$ and for all $j \in N \backslash M$.

Since $z_N \mathbf{R}(E) z'_N$ we have $|M| > 0$.And we have $|M| < |N|$ as $|M| = |N|$ contradicts with the Unanimous Strict Preference. Take $|M'| = |M| + 1$ and construct $R' \in \mathcal{R}^N$ such that $c(R'_i, q_i) < \min_{k \in N} c(R'_k, q_k) \leq c(R'_j, q_j)$ for all $i \in M$ and for all $j \in N \backslash M'$ and $q_N \mathbf{R}(E) q'_N$.

By repeating this construction $|N| - |M|$ times, we get a contradiction with the Unanimous Strict Preference.

Without loss of generality, we will take $1 \in M$, $2 \in N \backslash M$ and assume that $c(R_1, z_1) < c(R_2, z'_2) = \min_{k \in N} c(R_k, z'_k) < c(R_1, z'_1) < c(R_2, z_2)$.

So $((c_1, c_1), (c_2, c_2), z_{-12}) \mathbf{R}(E) ((c'_1, c'_1), (c'_2, c'_2), z'_{-12})$. As shown in Figure 4, by continuity and strict monotonicity, there exists $\varepsilon > 0$ such that $x_1(\varepsilon) < c_2 - \varepsilon$ and $x_2(\varepsilon) > c_2 - \varepsilon$ which ensures $(x_1(\varepsilon), c_2 - \varepsilon) I_1 (c_1, c_1)$ and $(x_2(\varepsilon), c_2 - \varepsilon) I_2 (c_2, c_2)$ and $x_1(\varepsilon) + x_2(\varepsilon) < c_2 - \varepsilon$. Then, there exist $y'(\varepsilon) > y(\varepsilon)$ and $x'_1(\varepsilon) < y'(\varepsilon)$ and $x'_2(\varepsilon) > y'(\varepsilon)$ which implies $(x'_1(\varepsilon), y'(\varepsilon)) I_1 (x_1(\varepsilon) + x_2(\varepsilon) + \varepsilon - c_2, c_2 - \varepsilon)$ and $(x'_2(\varepsilon), y'(\varepsilon)) I_2 (c'_2, c'_2)$ and $c_1 < y(\varepsilon) < y'(\varepsilon) < c'_2$

Now, we will choose $\varepsilon' > 0$ small enough to ensure that $(c_2, c_2) P_2 (x'_2(\varepsilon) +$

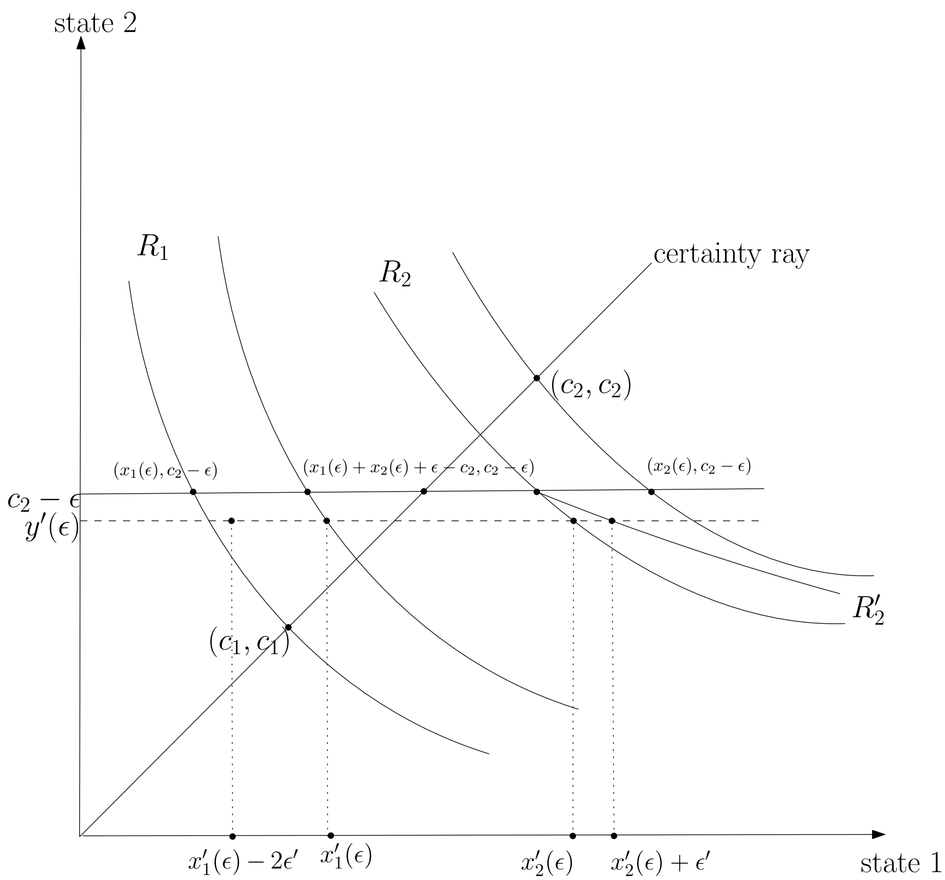

Figure 4: UI, R, and AAG forces CE Maximin ordering.

$\varepsilon', y'(\varepsilon))$. Construct a preference $R_2' \in \mathcal{R}$ such that $B(R_2', (c_2', c_2')) = B(R_2, (c_2', c_2'),\ (x_2'(\varepsilon) + \varepsilon', y'(\varepsilon))I_2'(c_2 - \varepsilon, c_2 - \varepsilon),\ (x_2(\varepsilon), c_2 - \varepsilon)I_2'(c_2, c_2)$.

Let $R_i' = R_i$, for all $i \in N \backslash \{2\}$ and let $q = ((x_1'(\varepsilon) + 2\varepsilon', y'(\varepsilon)), (x_2'(\varepsilon) - \varepsilon', y'(\varepsilon)), z_{-12}$.

$q_N \mathbf{P}(E')((x_1'(\varepsilon) + y'(\varepsilon)), (x_2'(\varepsilon) + \varepsilon', y'(\varepsilon)), z_{-12})$

$\mathbf{I}(E')((x_1(\varepsilon) + x_2(\varepsilon) + c_2 - \varepsilon, c_2 - \varepsilon), (c_2 - \varepsilon, c_2 - \varepsilon), z_{-12})$

$\mathbf{P}(E')((x_1(\varepsilon), c_2 - \varepsilon), (x_2(\varepsilon), c_2 - \varepsilon), z_{-12})$

$\mathbf{I}(E')((c_1, c_1), (c_2, c_2), z_{-12})$

$\mathbf{R}(E')((c_1', c_1'), (c_2', c_2'), z_{-12}) = q_N'$ by applying Aversion to Attendant Gains, Unanimous Indifference, Aversion to Attendant Gains, Unanimous Indifference and Responsiveness respectively.

Now, we take $M' = M \cup \{2\}$ and arrive at a contradiction by repeating these steps. □

4 Concluding Remarks

In this paper, we provide an axiomatic characterization of welfare egalitarianism defined by the certainty equivalence form. The equity condition formulated by the Aversion to the Attendant Gains axiom, which is a fairly minimal condition combined with Unanimous Indifference and Responsiveness, leads to an ordering which gives absolute priority to the worse off. By making use of ordinal and noncomparable preferences, and providing social orderings for all the possible preference profiles, this model is quite rich for policy analysis which seeks to recommend second-best allocations. For problems in which the policymaker has imperfect information on the individuals who are bounded by incentive constraints, efficient allocations might not be implementable. The social welfare ordering defined in this paper can give the second-best allocations by maximizing this ordering, subject to the relative constraints defined by that particular problem, such as status quo and incentive constraints. As for public policy examples, one can apply our social welfare criteria in optimal insurance design and in the mitigation of macroeconomic risks.

There are various resource equality axioms in the fair allocations literature such as Equal Split Transfer, Proportional Allocations Transfer, Equal Split Allocation, Transfer among Equals, and Nested Contour Transfer. Certainty Equivalent Leximin Ordering satisfies all of these axioms. One axiom stands out in the state contingent endowment framework: Proportional Allocations transfer in which proportionality is defined on the certainty ray. This axiom is weaker than the Aversion to the Attendant Gains axiom. It is interesting to study other robustness conditions weaker than Responsiveness, so that it forces social ordering to be in maximin form combined with Unanimous Indifference and Proportional Allocations transfer.

Social ordering in the leximin form can be seen as strongly egalitarian. There are other social ordering functions in the literature relaxing this strong form of egalitarianism. One example is the Nash-product social welfare function instead of the leximin criterion. This social ordering satisfies Pareto in a strong sense and the Proportional Allocations Transfer, but none of the aforementioned transfer axioms. For future research, we will study the possible characterization of Nash-product maximin ordering with appropriate robustness conditions.

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